Patterns in Geomorphology
Antonello Provenzale
IGG CNR, Italy
Hezi Yizhaq
BIDR, Ben Gurion University, Israel
Patterns of erosion and chemical dissolution due to running water from snow melting (Lapiaz)
Patterns of erosion and chemical dissolution
Erosion patterns in badlands (Tzin valley, Israel)
Patterns of erosion and deposition:
Meandering rivers
(Ucayali river, Peru)

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Patterns of erosion and deposition: Braided rivers
River networks
(Yemen)

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Coastal patterns (coast of Carolina, USA)

Courtesy of A. Brad Murray
Aeolian patterns
Great Sand Dunes National Monument, Colorado,
photo by Bob Bauer
Aeolian bedforms in deserts and sandy beaches:

Ripples
(length of cm, amplitude of mm)

Megaripples
(length of meters, amplitude of cm)

Dunes
(length of tens or hundreds of meters, amplitude of (tens of) meters)
Types of (non-vegetated) dunes:

- Barchans
- Transverse
- Seif (linear) dunes
- Star dunes
Barchan dunes on Mars

http://www.marsunearthed.com/SelectedImages/
Namib desert

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Linear dunes, Mauritania

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Namib sand sea

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Complex dune structures (Saudi Arabia)
Complex dune structures (Gran Desierto)

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Superposed aeolian bedforms

http://daac.gsfc.nasa.gov/DAAC_DOCS/geomorphology
Blown by wind:
Nonlinear dynamics of aeolian sand ripples
Properties of aeolian ripples:

Ripple index about 15-20

Almost 1D bedforms: crests are perpendicular to the wind, with defects

Slight asymmetry between lee and stoss slopes (no slip face)

Need of a wind intensity threshold to have ripple formation

Rapid response to wind

The ripple pattern coarsens with time and it slowly moves downstream

Grain sorting
A short history of ripple studies:

R.A. Bagnold, *The Physics of Blown Sand and Desert Dunes*, 1941

R.P. Sharp, *J. of Geology*, 1963

H. Nishimori and N. Ouchi, *PRL*, 1993

L. Prigozhin, *PRE*, 1999
Z. Csahok, C. Misbah, F. Rioual and A. Valance, *cond-mat*, 2000
H. Yizhaq et al. *EPSL*, 2019 (megaripples)
Mechanism of ripple formation: aeolian ripples form due to the instability of a flat sand bed exposed to strong wind
When the wind starts to blow, sand grains are lifted into the air. These grains are accelerated by the wind and fall down, hit the surface, and eject other grains.
The rebounding (saltating) grains are then accelerated by the wind, and a cascade process ensues.

An entire population of saltating grains emerges. The height of the saltation layer can be about one meter in strong winds.
Hypotheses of ripple formation (Cooke et al, 1993):

1. A rhythmic barrage of saltating grains (Bagnold)

2. The wave hypothesis:
   a) The bed as a fluid
   b) The saltation curtain as a fluid
   c) Wave-like instabilities in the boundary layer
   d) Secondary motions in the lee of transverse ripples

3. The role of reptating grains (Anderson 1987)
Saltation with typical jump length $L$
Reptation with typical jump length $a << L$
$L$ is about $1$ m and $a$ is about $1$ cm
Upon impact, the energy of a saltating grain goes as:
80% to one (on average) rebounding grain,
10% to a few reptating grains,
10% to the deformation of the bed
Depth of the saltation layer: up to about 1 m
Depth of the reptation layer: a few mm
We can idealize the problem in terms of a sand surface bombarded by a continuous flux of saltating grains that hit the surface at constant (small) angle $\phi = 8-12^\circ$.

The saltating grains drive the system. The important dynamics is contained in the behavior of the reptating grains.
Conservation of sand

\[(1 - n) \rho \frac{\partial \zeta}{\partial t} = -\nabla \cdot \mathbf{Q}\]

- \(n\) porosity of the bed (about 0.35)
- \(\rho\) density of sand
- \(\zeta\) elevation of the sand surface
- \(\mathbf{Q}\) flux of sand grains
The flux of sand:

\[ Q = Q_s + Q_r \]

- \( Q_s \) flux of saltating grains
- \( Q_r \) flux of reptating grains
The flux of saltating grains is assumed to be constant for aeolian ripples.

\[ \nabla \cdot Q_s = 0 \]

All the dynamics is contained in the variability of the reptation flux.

NB: This is untenable for megaripples and dunes.
There is no feedback of the aeolian bedforms on the wind and on the flux of saltating grains

NB: This is untenable for dunes
1D case

if all the grains had the same reptation length \( a \)

\[
Q_r^{bare}(x, t) = m \int_{x-a}^{x} N_{ej}(x') dx'
\]

- \( m \) mass of a sand grain
- \( N_{ej} \) number of ejected grains
- \( N_r \) average number of reptating grains ejected by one saltating grain
- \( N_{ej} = N_r N_{im} \) where \( N_{im} \) is the number of impacting grains
For a distribution of reptation lengths

\[ Q_{r}^{bare}(x, t) = m \int_{-\infty}^{x} d\alpha \ p(\alpha) \int_{-\infty}^{x} N_{ej}(x') \ dx' \]

- \( m \) mass of a sand grain
- \( N_{ej} \) number of ejected grains
- \( N_{r} \) average number of reptating grains ejected by one saltating grain
- \( N_{ej} = N_{r} N_{im} \) where \( N_{im} \) is the number of impacting grains
- \( p(\alpha) \) distribution of reptation lengths

\[ \int \alpha p(\alpha) d\alpha = a \]
Important angles

- wind direction
- saltating grain
- reptating grain
- stoss slope
- slope
- θ
- φ
- γ
Number density of impacting grains

\[ N_{im}(x) = N_{im}^0 \left( 1 + \frac{\tan \theta}{\tan \phi} \right) \cos \theta \]

\[ = N_{im}^0 \cot \phi \left( \frac{\tan \phi + \zeta_x}{\sqrt{1 + \zeta_x^2}} \right) \]

*\( N_{im}^0 \) number density of impacting grains on a horizontal surface (about \( 10^7 \text{ m}^{-2} \text{ s}^{-1} \))

*\( \theta \) inclination of the surface

*\( \tan \theta = \zeta_x \)
\[ a = 2 \frac{V^2}{g} \frac{\sin(\gamma - \theta) \cos \gamma}{\cos \theta} = a_{hor} (1 - \cot \gamma \tan \theta) \]
Shadowing:
the flux of reptating grains becomes

\[ N_{im}(x) = N_{im}^0 \cot \phi \ Max \left\{ \frac{\tan \phi + \zeta_x}{\sqrt{1 + \zeta_x^2}}, 0 \right\} \]
The full integral model:

\[
\frac{\partial \zeta}{\partial t} = -Q_0 \frac{\partial}{\partial x} \left[ (1 - \mu \zeta_x) \int_{-\infty}^{\infty} d\alpha \ p(\alpha) \int_{x-\alpha}^{x} F(x') \, dx' \right]
\]

\[
F(x) = \text{Max} \left\{ \frac{\tan \phi + \zeta_x}{\sqrt{1 + \zeta_x^2}}, 0 \right\}
\]

\[
Q_0 = \frac{mN_r N_{im}^0 \cot \phi}{\rho (1 - n)}
\]
Linear stability analysis

\[ Q_r(x,t) = Q_r^{\text{bare}}(x,t) \left( 1 - \mu \zeta_x \right) \]
Coarsening of the ripple pattern

The graph shows the relationship between the mean ripple wavelength (mm) and time (min). The data points are differentiated by the parameter \( \mu \) as indicated in the legend: + \( \mu=0.3 \), ○ \( \mu=0.5 \), × \( \mu=0.7 \), □ \( \mu=0.9 \), and ◇ \( \mu=1.1 \). The slope of the trend line is 0.35.

The x-axis represents time in minutes, ranging from 10^{-1} to 10^1, and the y-axis represents the mean ripple wavelength in millimeters, ranging from 10^0 to 10^2.
An extension to the 2D case:

If all the grains had the same jump length $a$,
the flux at $(x,y)$ in the direction $\psi$ to the $x$ axis
is assumed to be proportional to the number of grains ejected
between $(x - a \cos \psi, y - a \sin \psi)$ and $(x,y)$
Fig. 12. Snapshots of a two-dimensional ripple field produced by the numerical integration of Eq. (28) on a 256 x 256 grid with periodic boundary condition, at $T = 2, 4, 8, 16$. The wind direction is from left to right. Parameter values: $\epsilon = \mu = 0.25, \phi = 10^\circ, \beta_1 = \beta_3 = 1, \beta_2 = \tan \phi$ and a domain size of $5\pi/2$. 
Megaripples on Earth and Mars
The origin of the transverse instability of aeolian megaripples

H. Yizhaq, G. Bel, S. Silvestro, T. Elperin, I.F. Kok, M. Cardinale, A. Provenzale, and I. Katra

Field work
Laboratory experiments
Numerical simulations
Numerical simulations
Numerical simulations
Lateral flux dependent on height
Mars case
A much harder problem:
The dynamics of aeolian sand dunes
Just a beginning in the fascinating world of (eco)geomorphological pattern modelling